Supporting Information

“Going Into Government: How Hiring from Special Interests Reduces Their Influence”

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Table of Contents

A Formal Results A1
B Ideological Co-optation A12
C Potential Concerns A14

A Formal Results

The set up of the model is as in the main text of the paper. As we mention, we make several technical assumptions, which are formally stated in the following sections as they become relevant for the analysis. Because this is a sequential game of perfect information, we analyze via backward induction.

A.1 The Core Policy-Making Model

We analyze a version of the core policy-making subgame. Then, in the following subsection, we examine how entry affects the the equilibrium of this subgame.

Uninfluenced Policy. At any subgame where P pursues its own policy (i.e., after $a = 0$ or $p = 0$), its optimal policy is characterized as follows.

> **Lemma A1.** If $a = 0$, then P's best response is to set $x_P = \hat{x}_P$ and $q_P = h\hat{q}_P$, where $\hat{q}_P$ is strictly positive and solves $C'_P(\hat{q}_P) = b$.

Proof. If $a = 0$ or $p = 0$, then P chooses x_P and q_P to maximize the following (interim) utility:

$$h(bq_P - C_P(q_P)) - \alpha_P(\hat{x}_P - x_P)^2$$

Since this utility function is additively separable in x_P and q_P, we can examine each in turn. First, it is straight-forward to see that $x_P = \hat{x}_P$ is a best response. Second, the optimal q_P is found by maximizing the function with respect to q_P. If $h = 0$ then q_P is not a choice variable (or, alternatively, $C_P(q_P) = \infty$ for all $q_P > 0$). If $h = 1$, the first order condition is

$$b - C'_P(q_P) = 0$$

Since the second order condition is strictly negative (i.e., $-C''_P(q_P) < 0$), then the value of q_P that satisfies the first order condition is a maximum. Moreover, since $C'_P(0) = 0$ and $C'_P(q_P) > 0$ for all $q_P > 0$, there exists a unique $\hat{q}_P$ that solves $b - C'_P(\hat{q}_P) = 0$. □

One piece of notation will be useful in the following analysis. We label P's utility from its ideal level of quality (assuming it invests in capacity) as $\hat{v}_P \equiv b\hat{q}_P - C_P(\hat{q}_P)$.

Influenced Policy. Now we consider the proposal made by G. First, we suppose G is willing to make a proposal and formally characterize this proposal. In order for G's policy to be implemented, it must induce P to accept. Denote G's equilibrium choice of policy as (x_G^*, q_G^*). The following interim result allows us to bound the scope of our analysis.

Working backward, we next consider G's optimal proposal, assuming G finds it worthwhile to make an offer (which analyze below). Let $\hat{q}_G \geq 0$ be G's most preferred level of quality.

Lemma A2. Suppose G influences policy ($p = 1$). Then, for a given $h \in \{0, 1\}$, G's equilibrium policy is:

- $x_G^* = x_G^I$ and $q_G^* = \tilde{q}_G(x_G^I; h)$ if $\hat{q}_G \leq \tilde{q}_G(\hat{x}_G; h)$, where x_G^I is defined in the proof and $\tilde{q}_G(x_G; h) \equiv \frac{1}{b}\left(\alpha_P(x_G - \hat{x}_P)^2 + h\hat{v}_P\right)$, and
- $x_G^* = \hat{x}_G$ and $q_G^* = \hat{q}_G$ if $\hat{q}_G > \tilde{q}_G(\hat{x}_G; h)$.

Moreover, $x_G^* \in (\hat{x}_P, \hat{x}_G]$.

Proof. Assume G influences policy ($p = 1$). To induce acceptance, P must be better off with (x_G, q_G) than (x_P, q_P). Using Lemma A1, we can substitute equilibrium values for x_P and q_P and write the acceptance constraint:

$$bq_G - \alpha_P(x_G - \hat{x}_P)^2 \geq h\hat{v}_P - \alpha_P(\hat{x}_P - \hat{x}_P)^2$$

Rearranging yields: $q_G \geq \frac{1}{b}\left(\alpha_P(x_G - \hat{x}_P)^2 + h\hat{v}_P\right) \equiv \tilde{q}_G(x_G; h)$. Next, we find G's optimal proposal by first writing the Kuhn-Tucker Lagrangian:

$$\mathcal{L}(x_G, q_G, \mu) = bq_G - C_G(q_G) - \alpha_G(\hat{x}_G - x_G)^2 - \mu(\tilde{q}_G(x_G; h) - q_G)$$

The first order conditions are given by:

$$\begin{aligned} b - C_G'(q_G) + \mu &= 0 \\ 2\alpha_G(\hat{x}_G - x_G) - \mu\left(\frac{2\alpha_P}{b}(x_G - \hat{x}_P)\right) &= 0 \\ \mu(\tilde{q}_G(x_G; h) - q_G) &= 0 \end{aligned}$$

The third condition is the complementary slackness condition that can be satisfied with either $\mu = 0$ or $\tilde{q}_G(x_G; h) - q_G = 0$. We now consider each case.

- Assume $\mu = 0$. Then, solving the first order conditions yields the unique optimal policy proposal $q_G^C = \hat{q}_G$ and $x_G^C = \hat{x}_G$.
- Assume $q_G = \tilde{q}_G(x_G; h)$. Then, $\mu = C_G'(q_G) - b$, which we can substitute into the

second condition:

$$2\alpha_G(\hat{x}_G - x_G) - (C'_G(q_G) - b)\left(\frac{2\alpha_P}{b}(x_G - \hat{x}_P)\right) = 0$$

Rearranging yields the following, where the optimal policy proposal is uniquely defined by x_G^I and $\tilde{q}_G(x_G^I; h)$:

$$x_G^I = \frac{\alpha_G b \hat{x}_G + \alpha_P \hat{x}_P (C'_G(q_G) - b)}{\alpha_G b + \alpha_P (C'_G(q_G) - b)} \tag{1}$$

Since $C'_G(q_G) - b \geq 0$, then x_G^I is a weighted average of $\hat{x}_P$ and $\hat{x}_G$: $x_G^I \in (\hat{x}_P, \hat{x}_G]$.

Finally, we consider when each case arises. Using the complementary slackness condition, $x_G^* = x_G^C$ if $\hat{q}_G > \tilde{q}_G(\hat{x}_G; h)$, and $x_G^* = x_G^I$ otherwise. □

To limit the number of cases we need to consider (formally, to rule out corner solutions), we will assume that G cannot influence policy without offering concessions to P.

Assumption A1. G cannot induce P to accept G's most preferred policy:

$$\hat{q}_G < \frac{\alpha_P}{b}(\hat{x}_G - \hat{x}_P)^2 < \frac{\alpha_P}{b}(\hat{x}_G - \hat{x}_P)^2 + \frac{\hat{v}_P}{b}.$$

To make the following analysis tidy, we will make a strong version of this assumption, which will ensure that $\hat{q}_G = 0$.

Assumption A2. $C_G(q_G) = c_G q_G$, where $c_G > b$.

At a technical level, this more stringent assumption means that x_G^I is not a function of q_G. (Note: below, we still allow x_G^I to depend on entry, as will become apparent). In what follows, we will always use Assumption A1, however we will be explicit when we invoke Assumption A2.

Assumption A3. Assume that G influences policy ($p = 1$) when indifferent and that P invests in initial capacity ($h = 1$) when indifferent.

Special Interest Group Participation in Policy-Making. The foregoing analysis assumes that G influences policy. However, G may be better off with P's in-house policy than when it tries to influence policy. We now analyze when G influences policy-making.

Lemma A3. G influences policy if and only if P is low capacity ($h = 0$) or if P is high capacity ($h = 1$) and $\hat{x}_G \geq \hat{x}_G^{\text{p}}$, where $\hat{x}_G^{\text{p}}$ exists and is defined in the proof.

Proof of Lemma A3. Using Assumption A3, G influences policy when indifferent. Then consider the two cases defined by whether P is high or low capacity (i.e., $h = 1$ or $h = 0$). **Case L** ($h = 0$): G influences policy if and only if:

$$\underbrace{bq_G^* - C_G(q_G^*) - \alpha_G(\hat{x}_G - x_G^*)^2}_{u_G(x_G^*, q_G^*)} \geq \underbrace{-\alpha_G(\hat{x}_G - \hat{x}_P)^2}_{u_G(\hat{x}_P, 0)}$$

Note that $x_G = \hat{x}_P$ and $q_G = 0$ are feasible policy choices for G. However, by Lemma A2, G's optimal policy proposal entails $x_G^* > \hat{x}_P$ and $q_G^* \geq 0$, which by construction makes G better off than $x_G = \hat{x}_P$ and $q_G = 0$. Then, G is better off with its own policy proposal than P's policy when $h = 0$. **Case H** ($h = 1$): G influences policy if and only if:

$$\underbrace{bq_G^* - C_G(q_G^*) - \alpha_G(\hat{x}_G - x_G^*)^2}_{u_G(x_G^*, q_G^*)} \geq \underbrace{b\hat{q}_P - \alpha_G(\hat{x}_G - \hat{x}_P)^2}_{u_G(\hat{x}_P, \hat{q}_P)} \qquad \text{(PCH)}$$

Using Assumption A1 and Lemma A2, we can substitute $x_G^* = x_G^I$ and $q_G^* = \tilde{q}_G(x_G^I; h)$. Taking the derivative of each side of (PCH) with respect to $\hat{x}_G$ (and using the envelope theorem for the left hand side) yields:

$$\frac{du_G(x_G^I, \tilde{q}_G(x_G^I; h))}{d\hat{x}_G} = -2\alpha_G(\hat{x}_G - x_G^I) < 0 \qquad \frac{du_G(\hat{x}_P, \hat{q}_P)}{d\hat{x}_G} = -2\alpha_G(\hat{x}_G - \hat{x}_P) < 0$$

Both sides of (PCH) are decreasing in $\hat{x}_G$. However, since $-2\alpha_G(\hat{x}_G - \hat{x}_P) < -2\alpha_G(\hat{x}_G - x_G^I)$, the left hand side is decreasing in $\hat{x}_G$ more slowly than the right hand side. Moreover, at $\hat{x}_G = \hat{x}_P$, the condition fails. Together, this implies that there exists some $\hat{x}_G^{\text{p}} > \hat{x}_P$ such that (PCH) binds and that for all $\hat{x}_G \geq \hat{x}_G^{\text{p}}$, influencing policy is a best response for G. □

Investment in Capacity. Next, we consider whether P invests in initial capacity.

Lemma A4. P invests in initial capacity ($h = 1$) if and only if $k \leq \hat{v}_P$.

Proof of Lemma A4. Using Assumption A3, P invests in initial capacity when indifferent. Denote the equilibrium proposal of G with and without investment as (x_G^H, q_G^H) and (x_G^L, q_G^L), respectively. There are two case to consider. **Case 1**: Suppose that G influences policy regardless of whether P invests. P invests if and only if:

$$bq_G^H - \alpha_P(x_G^H - \hat{x}_P)^2 - k \geq bq_G^L - \alpha_P(x_G^L - \hat{x}_P)^2$$
$$\iff k \leq b(q_G^H - q_G^L) + \alpha_P(x_G^L - \hat{x}_P)^2 - \alpha_P(x_G^H - \hat{x}_P)^2 \qquad \text{(IC1)}$$

Now, we consider the form of x_G^H and q_G^H. Using Assumption A1, there is an interior solution regardless of whether there is investment. Then substituting $q_G = \tilde{q}_G(x_G^I; h)$, (IC1) reduces to $k \leq \hat{v}_P$. **Case 2**: Suppose that G participates if and only if P does not invest. P invests if and only if:

$$\hat{v}_P - k \geq bq_G^L - \alpha_P(x_G^L - \hat{x}_P)^2$$
$$\iff k \leq \hat{v}_P - (bq_G^L - \alpha_P(x_G^L - \hat{x}_P)^2) \tag{IC2}$$

Using Assumption A1, there is an interior solution regardless of whether there is investment. Then substituting $q_G = \tilde{q}_G(x_G^I; h)$, (IC2) again reduces to $k \leq \hat{v}_P$. □

A.2 Industry Insider's Entry into Government

We now introduce the possibility that I may enter government. Assume that entry affects *capacity*. As we describe in the main text, in this case, entry affects the players' costs from producing policy quality. Given I's entry choice ($e \in \{0, 1\}$) can rewrite their cost functions as:

$$\Psi_P(q_P, e, \pi) = [(1-e) + e(1-\pi)]C_P(q_P) \qquad \Psi_G(q_G, e, \gamma) = [(1-e) + e\gamma]C_G(q_G)$$

where $\pi \in [0, 1)$, $\gamma \geq 1$, and C_G and C_P are the players' "baseline" cost functions. We first restate Lemma A3 and Lemma A4 in terms of π.

Lemma A5. P invests in initial capacity ($h = 1$) if and only if $\pi \geq \overline{\Delta}^h$ where $\overline{\Delta}^h > 0$ exists and is implicitly defined by $\hat{v}_P(\overline{\Delta}^h) = k$.

Proof. Lemma Lemma A4 demonstrates that there is entry if $\hat{v}_P \geq k$. Since $\hat{v}_P$ is strictly increasing in π from 0 to ∞ on the interval $(-\infty, 1]$, so as long as $k > 0$, then there exists a threshold $\overline{\Delta}^h < 1$ where the condition binds and strictly holds for all $\pi > \overline{\Delta}^h$. Then, $\overline{\Delta}^h$ is implicitly defined as $\hat{v}_P(\overline{\Delta}^h) = k$, and we have therefore demonstrated that for all $\pi \geq \overline{\Delta}^h$, P invests in capacity. □

Lemma A6. G influences policy if P is low capacity, or if P is high capacity and $\pi \leq \overline{\Delta}^p(\gamma) < 1$, where $\overline{\Delta}^p(\gamma)$ is implicitly defined in the proof.

Proof. The first part of the result—that G influences policy if P is low capacity—is shown in the proof of Lemma A3 above. Now consider the case where P is high capacity. Using

Assumption A3, G influences policy when indifferent. Then, G influences policy if:

$$\underbrace{bq_G - \Psi_G(q_G, e, \gamma) - \alpha_G(\hat{x}_G - x_G)^2}_{u_G(x_G^*, q_G^*)} \geq \underbrace{b\hat{q}_P - \alpha_G(\hat{x}_G - \hat{x}_P)^2}_{u_G(\hat{x}_P, \hat{q}_P)} \tag{PCH$'$}$$

where we substitute Ψ_G for C_G. By Assumption A1, participation entails interior solutions, so we can substitute $x_G^* = x_G^I$ and $q_G^* = \tilde{q}_G(x_G^I; h)$. Taking the derivative of each side of (PCH′) with respect to π (and using the envelope theorem for the left hand side) yields:

$$\frac{du_G(x_G^I, q_G^I)}{d\pi} = \frac{1}{b}\frac{\partial \hat{v}_P}{\partial \pi}\left(b - \Psi_G'(q_G^I(e), e, \gamma)\right) < 0 \qquad \frac{du_G(\hat{x}_P, \hat{q}_P)}{d\pi} = b\frac{\partial \hat{q}_P}{\partial \pi} > 0$$

Then, the left hand side of (PCH′) is monotonically decreasing in π and the right hand side is monotonically increasing in π. Moreover, since $\hat{q}_P \to \infty$ as π approaches 1 and $\hat{q}_P \to 0$ as π approaches $-\infty$, then the right hand side increases from 0 to ∞. Then, there exists some $\overline{\Delta}^p(\gamma) < 1$ such that for all $\pi \leq \overline{\Delta}^p(\gamma)$, participation is a best response for G. □

We wish to focus attention on the most interesting case where G influences policy at least some of the time. So, we assume that without entry (i.e., when G is at its strongest), there is participation.

Assumption A4. Without entry, G has a strict incentive to participate. Formally, $\overline{\Delta}^p(\gamma = 1) > 0$.

We now denote all endogenous parameters as explicit functions of of e.

Lemma A7. I's entry into government has the following effect on the ideological content of G's policy proposal if $p = 1$:

- If C_G is linear in q_G or $h(1) = 0$ and if $\gamma = 1$, then G's equilibrium proposal is ideologically unchanged after entry: $\hat{x}_P < x_G^I(e = 1) = x_G^I(e = 0) < \hat{x}_G$.
- Otherwise, then G's equilibrium proposal becomes more ideologically favorable to P after entry: $\hat{x}_P < x_G^I(e = 1) < x_G^I(e = 0) < \hat{x}_G$.

Moreover, if Assumption A2 holds and $\gamma > 1$, then entry always moderates G's influence over policy: $x_G(e = 1) < x_G(e = 0)$.

Proof. If G influences policy, using Assumption A1, then there is always an interior solution (regardless of entry), so $x_G(e)$ is given by:

$$x_G^I(e) = \frac{\alpha_G b\hat{x}_G + \alpha_P\hat{x}_P([(1-e) + e\gamma]C_G'(q_G) - b)}{\alpha_G b + \alpha_P([(1-e) + e\gamma]C_G'(q_G) - b)} \tag{2}$$

which is strictly between $\hat{x}_P$ and $\hat{x}_G$ using Assumption A1.

Case 1. Suppose C_G is linear in q_G. Then $C'_G(q_G) = C'_G(q_G)$ for all q_G. Then, it is straight-forward to see from (2) that $x^I_G(1) = x^I_G(0)$ if $\gamma = 1$ and $x^I_G(1) < x^I_G(0)$ if $\gamma > 1$.

Case 2. Suppose C_G is strictly convex in q_G and $h(1) = 0$. Then, $q_G(1) = q_G(0)$ and it is straight-forward to see from (2) that $x^I_G(1) = x^I_G(0)$ if $\gamma = 1$ and $x^I_G(1) < x^I_G(0)$ if $\gamma > 1$.

Case 3. Suppose C_G is strictly convex in q_G and $h(1) = 1$. Then, $q_G(1) > q_G(0)$ and by the strict convexity of C_G, $\gamma C'_G(q_G(1)) > C'_G(q_G(0))$ for all $\gamma \geq 1$. Then, from (2) it follows that $x^I_G(1) < x^I_G(0)$.

Finally, if Assumption A2, then C_G is linear in q_G and so if $\gamma > 1$, $x_G(1) < x_G(0)$. □

Lemma A8. *I's entry into government has the following effects on P and G:*

- *If $\pi \geq \overline{\Delta}^h > 0$, entry induces P to invest in initial capacity.*
- *If $\pi \geq \overline{\Delta}^h$ and $\pi > \overline{\Delta}^p(\gamma)$, entry induces G to stop influencing policy.*

Proof. The first result follows from the fact that $\overline{\Delta}^h > 0$ implies P does not invest before entry while $\pi \geq \overline{\Delta}^h$ implies it does invest after entry. The second result follows directly from the fact that $\overline{\Delta}^p(\gamma) < 1$. Thus, there exist some π sufficiently close to 1 such that $\pi \geq \overline{\Delta}^h$ but $\pi > \overline{\Delta}^p(\gamma)$. □

Lemma A9. *If $\pi > 0$, then entry makes P better off and G worse off. Moreover, P is increasingly better off and G is increasingly worse off with entry as π increases.*

Proof. Suppose $\pi > 0$. First note that this implies $\hat{v}_P(e = 1) > \hat{v}_P(e = 0)$. Next, suppose $p = 1$ for all $e \in \{0, 1\}$. By Assumption A1, there is an interior solution and q_G is set to make P indifferent: $u_P(x_G, q_G) = bq_G - \alpha_P(x_G - \hat{x}_P)^2 = h\hat{v}_P = u_P(x_P, q_P)$. Since $\hat{v}_P(e = 1) > \hat{v}_P(e = 0)$, it follows that $u_P(x_G, q_G)$ must also weakly increase. Finally, suppose $p = 1$ if and only if $e = 0$. This implies after entry, P invests since G always participates if P does not invest (see Lemma A3). Then, it is straight forward to see that:

$$\hat{v}_P(1) > b\left[\frac{1}{b}\alpha_P(x^I_G(0) - \hat{x}_P)^2 + h\hat{v}_P(0)\right] - \alpha_P(x^I_G(0) - \hat{x}_P)^2 \iff \hat{v}_P(1) > h\hat{v}_P(0)$$

Finally, it is straight-forward to see P is increasingly better off with entry as π increases since

$\hat{v}_P$ increases in π.

Now, we show that G is worse off. Using the envelope theorem, it follows that $u_G(x_G^I, q_G^I)$ is decreasing in π: $du_G(x_G^I, q_G^I)/d\pi = -(1/b)(\partial \hat{v}_P/\partial \pi)(\Psi_G'(q_G^I) - b) < 0$. Since entry increases π, G is worse off. Finally, it is straight-forward to see G is increasingly worse off with entry as π increases since $u_G(x_G^I, q_G^I)$ is decreasing in π. □

A.3 Industry Insider's Incentive to Go into Government

Assumption A5. Assume that I does not enter government ($e = 0$) when indifferent.

Let $x^*(e)$ and $q^*(e)$ be the equilibrium policy outcomes as a function of e. Using Assumption A5, endogenous entry by I requires:

$$bq^*(1) - \alpha_I(\hat{x}_G - x^*(1))^2 > bq^*(0) - \alpha_I(\hat{x}_G - x^*(0))^2$$
$$\iff q^*(1) - q^*(0) > \frac{\alpha_I}{b}\left[(\hat{x}_G - x^*(1))^2 - (\hat{x}_G - x^*(0))^2\right] \quad \text{(EC)}$$

The analysis in the prior subsection demonstrates that entry affects (1) whether G influences policy, and (2) whether P invests in initial capacity. We now consider several mutually exclusive and exhaustive cases.

First, suppose P does not invest in capacity regardless of whether I enters government.

Lemma A10 (weak policy-maker)**.** Suppose $0 < \pi < \overline{\Delta}^h$. Then, given Assumption A5, entry is never a best response.

Proof. Suppose that $0 < \pi < \overline{\Delta}^h$. Then from Lemma A3, G influences policy for all e, and using Assumption A1: $q_G^*(e) = \tilde{q}_G(x_G^I(e); 0) = \alpha_P(x_G^I(e) - \hat{x}_P)^2/b$. Moreover, from Lemma A7, $x_G^I(1) \leq x_G^I(0)$. First, if $x_G^I(1) < x_G^I(0)$, then the condition fails since the right hand side is strictly positive whereas the left hand side is strictly negative. Second, if $x_G^I(1) = x_G^I(0)$, then the condition fails (using Assumption A5) since the right hand side and left hand side are both zero. Thus, entry is not a best response. □

Second, suppose P invests after entry, but G shuts down.

Lemma A11 (group shut down)**.** Suppose that $\pi \geq \overline{\Delta}^h$ and $\pi > \overline{\Delta}^p(\gamma)$, then entry is a best response if and only if $\pi > \widetilde{\Delta}^{\text{gsd}}_{h(0)}$, where $\widetilde{\Delta}^{\text{gsd}}_{h(0)}$ is defined in the proof.

Proof. Assume $\pi \geq \overline{\Delta}^h$ and $\pi > \overline{\Delta}^p(\gamma)$. Given that G influences policy if and only if there is no entry, and using Assumption A1, I will find it optimal to enter if:

$$\hat{q}_P(1) - \tilde{q}_G(x_G^I(0); h) > \frac{\alpha_I}{b}\left((\hat{x}_G - \hat{x}_P)^2 - (\hat{x}_G - x_G^I(0))^2\right)$$

Let $h(0)$ indicate whether P invests before entry. Then, this can be written

$$\hat{q}_P^1 > \frac{\alpha_I}{b}\left((\hat{x}_G - \hat{x}_P)^2 - (\hat{x}_G - x_G^I(0))^2\right) + \frac{1}{b}(\alpha_P(x_G^I(0) - \hat{x}_P)^2 + h(0)\hat{v}_P(0)) \qquad \text{(ECN)}$$

On the interval $[\overline{\Delta}^h, 1]$, which is the relevant interval under consideration here, the left hand side strictly increases in π from some real-valued number to ∞. Then, since the right hand side is a constant real valued (positive) number, there is some threshold $\widetilde{\Delta}^{\text{gsd}}_{h(0)} \in [\overline{\Delta}^h, 1)$ such that for all $\pi > \widetilde{\Delta}^{\text{gsd}}_{h(0)}$, (EC) holds and there is entry. □

Finally, suppose that P invests after entry and G influences policy after entry. We call this case "competitive influence." This case may or may not arise in an equilibrium of the baseline policy-makinhg model. Before proceeding, we need to establish when this case will arise.

Definition A1. There is *competitive influence* if and only if $\pi \in [\overline{\Delta}^h, \overline{\Delta}^p(\gamma)]$.

Lemma A12. Using Assumption A2, competitive influence is feasible if $k < \tilde{k}$ and $\gamma < \tilde{\gamma}(k)$, where $\tilde{k}$ and $\tilde{\gamma}(k)$ are defined in the proof.

Proof. We seek to characterize when $\overline{\Delta}^h < \overline{\Delta}^p(\gamma)$. First, from Lemma A5, it follows that $\overline{\Delta}^h$ is increasing in k. So, we can write the condition as:

$$\overline{\Delta}^h(k) < \overline{\Delta}^p(\gamma)$$

Next, note that $\overline{\Delta}^p(\gamma)$ is decreasing in γ. So, there exists some $\tilde{k}$ such that $\overline{\Delta}^h(\tilde{k}) = \overline{\Delta}^p(1)$. And as long as $k < \tilde{k}$, there exists some $\tilde{\gamma}$ such that $\overline{\Delta}^h(k) = \overline{\Delta}^p(\tilde{\gamma})$. Taking these facts together: for all $k < \tilde{k}$ and all $\gamma < \tilde{\gamma}(k)$, $\overline{\Delta}^p(\gamma) < \overline{\Delta}^h$. Otherwise, $\overline{\Delta}^p(\gamma) \geq \overline{\Delta}^h$. □

Lemma A13 (competitive influence)**.** Suppose competitive influence is feasible and that Assumption A2 holds. Let $h(0)$ indicate P's investment decision if I does not enter. Then, if $\max\{\overline{\Delta}^h, \widetilde{\Delta}^{\text{ci}}_{h(0)}(\gamma)\} \leq \pi \leq \overline{\Delta}^p(\gamma)$ and $\gamma < \overline{\gamma}_{h(0)}$ (where $\overline{\gamma}_{h(0)} > 1$), then entry is a best response. Moreover, $\widetilde{\Delta}^{\text{ci}}_0(\gamma) < \widetilde{\Delta}^{\text{ci}}_1(\gamma)$.

Proof. Assume that $\pi \geq \overline{\Delta}^h$ and $\pi \leq \overline{\Delta}^p(\gamma)$. Using Lemma A10, we only consider the cases in which $h(1) = 1$. Fix $h(0)$ to indicate P's capacity before entry. Using Assumption A2, we can substitute and write (EC) as:

$$\alpha_P \left((\hat{x}_P - x^I_G(1,1))^2 - (\hat{x}_P - x^I_G(h(0),0))^2\right) + \hat{v}_P(1) - h(0)\hat{v}_P(0)$$
$$> b\frac{\alpha_I}{b}\left((\hat{x}_G - x^I_G(1,1))^2 - (\hat{x}_G - x^I_G(h(0),0))^2\right) \tag{ECP}$$

Again, using Assumption A2, it follows that $x^I_G(\cdot)$ is constant in π but declining in γ. Then, $x^I_G(h, e = 1) < x^I_G(h, e = 0)$ if $\gamma > 1$. For a fixed γ, the left hand side is increasing in π from some weakly positive real number to ∞, and the right hand side is constant in π. So, fixing all other parameters, there exists some threshold $\widetilde{\Delta}^{\text{ci}}_{h(0)}(\gamma)$, which is a function of γ such that for all $\pi > \widetilde{\Delta}^{\text{ci}}_{h(0)}(\gamma)$, the condition holds. Moreover, as γ increases, the left hand side decreases and the right hand side increases. This makes (ECP) less slack, implying that $\widetilde{\Delta}^{\text{ci}}_{h(0)}(\gamma)$ is increasing in γ.

Finally, we must derive the condition such that $\overline{\Delta}^p(\gamma) > \widetilde{\Delta}^{\text{ci}}_{h(0)}(\gamma)$. First, note that $\overline{\Delta}^p(\gamma)$ is increasing in γ from some $\overline{\Delta}^p(\gamma = 1) > 0$ (using Assumption A4). Second, note that at $\gamma = 1$, (ECP) reduces to

$$\frac{\hat{v}_P(1) - h(0)\hat{v}_P(0)}{b} > 0$$

(This last statement uses Assumption A2.) Thus, the condition (ECP) holds for all $\pi \in (0, 1)$ if $\gamma = 1$. Taking all this together, there exists some $\overline{\gamma}_{h(0)} > 1$ implicitly defined by $\overline{\Delta}^p(\overline{\gamma}_{h(0)}) = \widetilde{\Delta}^{\text{ci}}_{h(0)}(\overline{\gamma}_{h(0)})$ such that for all $\gamma < \overline{\gamma}_{h(0)}$, $\widetilde{\Delta}^{\text{ci}}_{h(0)}(\gamma) < \overline{\Delta}^p(\gamma)$.

We now prove the final part of the result. Examining (ECP), the only term that depends on $h(0)$ is $h(0)\hat{v}_P(0)$. Note that if $h(0) = 1$, then the left hand side decreases, making the condition less slack. Since $\widetilde{\Delta}^{\text{ci}}_{h(0)}$ is defined by the π where the condition binds, and using the logic from above, it follows that $\widetilde{\Delta}^{\text{ci}}_0 < \widetilde{\Delta}^{\text{ci}}_1$. We have proved the result. □

Lemma A14. Entry is optimal for a wider range of parameter values as α_I decreases. At $\hat{x}_I = \hat{x}_G$, a marginal decrease in $\hat{x}_I$ makes entry optimal for a wider range of parameter values.

Proof. Examining (ECN) and (ECP), which implicitly define $\widetilde{\Delta}^{\text{gsd}}_{h(0)}$ and $\widetilde{\Delta}^{\text{ci}}_{h(0)}$, respectively, it is straight-forward to see that each cutoff increases in α_I for all $\alpha_I > 0$. Next, consider how the thresholds change with a marginal decrease in $\hat{x}_I$ (from $\hat{x}_I = \hat{x}_G$). It is again straight-forward to see that each cutoff increases in $\hat{x}_I$ for all $\hat{x}_I \geq x^I_G(h(0); 0)$, since I's ideological losses from entry decrease but her quality gain remains unchanged. So, both (ECN) and (ECP) hold for a wider range of α_Is as $\hat{x}_I$ decreases from $\hat{x}_G$. □

Lemma A15. Fix γ. Then, I's utility benefit from entering government in the group shut down case and in the competitive influence case increases as π increases.

Proof. The first claim follows directly from the observation in the proofs of Lemma A11 and Lemma A13 that the entry condition slackens in π. □

Proposition A1. There is a unique equilibrium of the model in which I enters government if $\pi \geq \overline{\Delta}^h$ and

1. $\pi > \max\{\widetilde{\Delta}^{\text{gsd}}_{h(0)}, \overline{\Delta}^p(\gamma)\}$, or
2. $k < \tilde{k}$, $\gamma < \min\{\tilde{\gamma}(k), \overline{\gamma}_{h(0)}\}$ and $\widetilde{\Delta}^{\text{ci}}_{h(0)} < \pi \leq \overline{\Delta}^p$.

Otherwise, there is a unique equilibrium in which I does not enter government.

Proof. Existence of an equilibrium follows directly from the previous results. Uniqueness follows from application of Zermelo's Theorem, and Assumption A3 and Assumption A5, which rule out the players' indifference at terminal nodes of the game. □

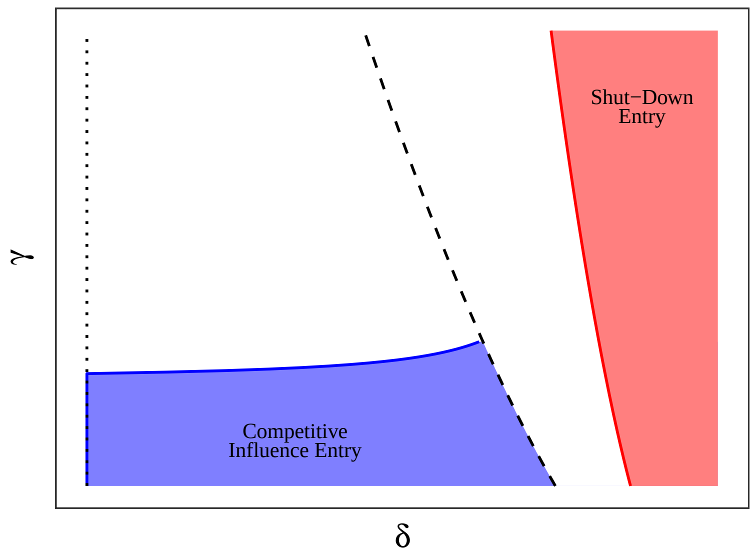
A.4 Example: Relaxing Assumption A2

If we were to relax Assumption A2 (while retaining Assumption A1 and Assumption A4), we could still get entry, but it is more difficult to characterize analytically. The following example will demonstrate that our main qualitative conclusions do not change.

Suppose that $\Psi_G(q_G) = \gamma(c_G/2)q_G^2$ and that Assumption A1 still holds. Moreover, we will fix values for the exogenous parameters so that the utility functions of the players are:

$$u_P = q - \frac{\pi}{2}q_P^2 - (x-0)^2 - \frac{h}{2} \qquad u_G = q - \frac{\gamma c_G}{2}q_G^2 - (x-2)^2 \qquad u_I = q - (x-2)^2$$

Figure 4: *We depict when there is entry in the example. The dashed line plots* $\overline{\Delta}^p(\gamma)$ *(for different values of* γ*) and the dotted line plots* $\overline{\Delta}^h$*. For a fixed* γ*, the shaded red region demonstrates the range of* π *where there is entry with group shut down, while the shaded blue region demonstrates the range of* π *where there is entry with competitive influence.*



Note that: $b = 1$, $\alpha_I = \alpha_G = \alpha_P = 1$, $c_P = 1$, $k = 1/2$, and $\hat{x}_P = 0 < 2 = \hat{x}_G = \hat{x}_I$. An interior equilibrium policy offered by G is the $x_G = x_G^I$ that maximizes is objective function, and a corresponding $q_G = \tilde{q}_G(x_G^I; h)$. Note that $\hat{q}_G = 1/(2\gamma c_G)$, so assumption A1 requires:

$$1/(2\gamma c_G) < \tilde{q}_G(x_G^I; h)$$

As long as c_G is sufficiently high, then there will always be an interior solution. Investment by P requires that $\frac{1}{2(1-\pi)} \geq \frac{1}{2} \iff \pi \geq 0 = \overline{\Delta}^h$. Conveniently, this ensures P invests before and after entry. For different values of γ, we can numerically calculate $\overline{\Delta}^p(\gamma)$ and $\widetilde{\Delta}^{\text{gsd}}_{h(0)}$, as well as the values of π for which there will be entry. We depict this in Figure 4.

B Ideological Co-optation

We now explore an alternative version of the model in which the industry insider's entry into government affects the policy-maker's ideal point. We call this "ideological co-optation."

Formally, when I enters government, this biases P to be more favorable to G by an amount $g > 0$ such that P's ideal point is $\hat{x}_P + eg$, and its policy loss from policy x is $-\alpha_P(\hat{x}_P + eg - x)^2$.

In the previous analysis, we implicitly assumed that P's policy preferences were perfectly aligned with the public, who we'll conceptualize as a representative "voter" and label V. However, now that I can affect P's ideal point, there is the possibility of divergence between the voter and P. We denote the V's ideal point as $\hat{x}_V$ and write its utility function as:

$$u_V(x, q) = bq - \alpha_V(x - \hat{x}_V)^2$$

Note that we assume the voter does not bear the cost of producing quality.

Lemma A16. With ideological co-optation, if G influences policy, then G's proposal becomes less ideologically favorable to V after entry. Formally, $\hat{x}_V < x_G(0) < x_G(1) < \hat{x}_G$. Moreover, $q_G(1) < q_G(0)$.

Proof. Examining (1), it is obvious that if x_G^I increases as $\hat{x}_P$ increases with ideological co-optation. Moreover,

$$\frac{1}{b}(\alpha_P(\hat{x}_P + g - x_G^I(1))^2 - h\hat{v}_P) < \frac{1}{b}(\alpha_P(\hat{x}_P - x_G^I(0))^2 - h\hat{v}_P)$$

So, $q_G(1) < q_G(0)$. □

Lemma A17. With ideological co-optation, entry makes G better off and V worse off if and only if $\hat{q}_P < \frac{1}{b}(\alpha_V g^2 + h(0)\hat{v}_P(0))$, where $h(0)$ indicates whether P is high capacity before entry.

Proof. We begin by considering V's utility. There are two cases to consider: G influences policy before and after entry, and G influences policy before, but not after, entry. First, suppose G influences policy before and after entry. By Lemma A16 (which uses Assumption A1), note that $x_G(1) > x_G(0) > \hat{x}_V$ and $q_G(1) < q_G(0)$. Then it is straight forward to see that V is worse off after entry since:

$$bq_G(1) - \alpha_V(\hat{x}_V - x_G(1))^2 < bq_G(0) - \alpha_V(\hat{x}_V - x_G(0))^2$$

Next, suppose G participates before but not after entry. Then we seek to show that

$$b\hat{q}_P - \alpha_V(\hat{x}_V - (\hat{x}_P + g))^2 < bq_G(0) - \alpha_V(\hat{x}_V - x_G^I(0))^2$$

Rearranging, substituting and simplifying yields:

$$\hat{q}_P < \frac{1}{b}(\alpha_V g^2 + h(0)\hat{v}_P)$$

Now, we show that G is better off. We do so by noting that by Assumption A1, G is always producing more quality than is optimal for G whenever it participates. In this case, since entry reduces the required quality for acceptance, this directly improves G's utility. Moreover, G is also able to craft a policy closer to its ideal point. G is unambiguously better off. Moreover, if entry induces G not to participate, it is because P's policy makes G better off than its own policy, which itself makes G better off after entry by the logic above. □

C Potential Concerns

C.1 Commonly Valued Quality

We now explore what happens as we vary the players' marginal benefit from quality, which we now index by $i \in \{P, G, I\}$ and allow to take different values. We continue to assume $b_P > 0$, since P valuing the q dimension of policy is a prerequisite for our model given that we constrain q to be non-negative.[9]

First, we examine how relaxing commonly valued quality affects the relationship between P and G, and specifically, whether G still seeks to influence policy. We focus on the case where $h = 1$ since Lemma A3 demonstrates G always influences policy when $h = 0$.

Lemma A18. Let $h = 1$. Using Assumption A2, then for any constellation of parameter values, there exists some $\hat{x}'_G(\cdot)$ such that for all $\hat{x}_G > \hat{x}'_G(\cdot)$, G influences policy. Moreover, if G values quality exactly opposite to P (i.e., $b_G = -b_P$), then G influences policy for a wider range of parameter values.

Proof. Suppose $h = 1$. From Lemma A3, recall that G influences policy if and only if $\hat{x}_G \geq \hat{x}^{\text{p}}_G$. Using Assumption A2, we can analytically characterize $\hat{x}^{\text{p}}_G$:

$$\hat{x}^{\text{p}}_G = \begin{cases} \hat{x}_P & \text{if } b_G < -c_G\gamma \\ \hat{x}_P + \dfrac{1}{2}\sqrt{\dfrac{(c_G\gamma + b_G)(\alpha_G b_P + \alpha_P(c_G\gamma - b_G))}{\alpha_G^2 c_P(1-\pi)}} & \text{if } b_G > -c_G\gamma \end{cases}$$

Taking the derivative of the bottom expression with respect to b_G yields an expression that is positive if $b_G < \frac{\alpha_G b_P}{2\alpha_P}$ and negative otherwise. Then, $\hat{x}^{\text{p}}_G$ is single-peaked with a maximum at

9. In principle, we can relax this if we allow q to take strictly negative values, but this does not provide qualitatively different insights.

$b_G = \frac{\alpha_G b_P}{2\alpha_P} \equiv \bar{b}_G$. At the maximum:

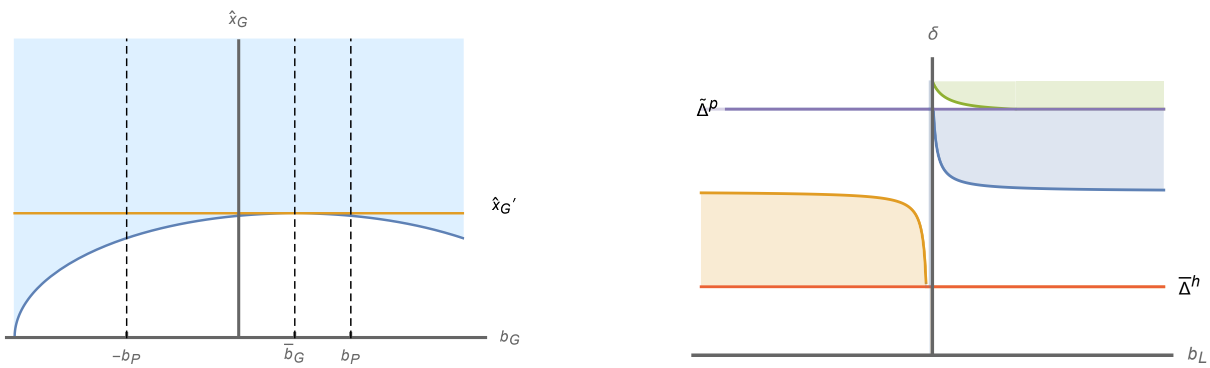
$$\hat{x}_G^{\text{p}}(b_G = \bar{b}_G) = \hat{x}_P + \frac{1}{4}\sqrt{\frac{(\alpha_G b_P + 2\alpha_P c_G \gamma)^2}{\alpha_G^2 \alpha_P c_P (1-\pi)}} \equiv \hat{x}_G' > \hat{x}_P$$

So, for all $b_G \in \mathbb{R}$, if $\hat{x}_G > \hat{x}_G'(\cdot)$ (for fixed parameters), then G influences policy.

Finally, it is straight forward to see that $\hat{x}_G^{\text{p}}(b_G = b_P) > \hat{x}_G^{\text{p}}(b_G = -b_P)$ by substituting for b_G in the expression for $\hat{x}_G^{\text{p}}$. □

We depict this result in the left panel of Figure 5.

Figure 5: *Left panel: We plot $\hat{x}_G^p$ as a function of b_G, fixing the other parameter values. G influences P in the blue shaded region. Note that G influences policy more often when perfectly opposed to P on the quality dimension (i.e., when $b_G = -b_P$) than when perfectly aligned (i.e., when $b_G = b_P$). Right panel: We plot $\overline{\Delta}^p$ as a function of b_I, fixing the other parameter values. I enters government in each of the shaded regions. The green region depicts entry in the group shut down case. The blue and orange regions depict entry in the competitive influence case for $b_I > 0$ and $b_I < 0$, respectively. The red and purple lines are $\overline{\Delta}^h$ and $\overline{\Delta}^p$, respectively.*



Next we examine how entry incentives change as we vary b_I.

Lemma A19. Suppose that $b_P, b_G > 0$. Then, (1) entry is more common as $|b_I|$ increases, and (2) if $b_I > 0$, entry occurs for π sufficiently high, and (2) if $b_I < 0$, entry occurs for π sufficiently low.

Proof. Examining (ECN) and (ECP), which implicitly define $\widetilde{\Delta}_{h(0)}^{\text{gsd}}$ and $\widetilde{\Delta}_{h(0)}^{\text{ci}}$, respectively, it is straight-forward to see that: (1) each cutoff decreases in b_I for all $b_I \in \mathbb{R}$ (except at asymptotes), and (2) entry occurs if $\pi > \widetilde{\Delta}^{\text{gsd}}_{h(0)}$ when $b_I > 0$, but when $\pi < \widetilde{\Delta}^{\text{gsd}}_{h(0)}$ when $b_I < 0$. □

We depict this result in the right panel of Figure 5.

C.2 Strong Entry

We modify the game slightly to add the option for I to choose whether to exit government after entering. Since we are interested in examining a commitment problem on the part of I (see text), we make the simplifying assumption that she can choose to return to G, $r \in \{0, 1\}$, after P decides whether to invest in capacity. Before we proceed, note that this may advantage the I, who can use her employment decisions to get P to invest, and then go back to G to get better policy outcomes. This is why we consider it.

Proposition A2. Suppose that competitive influence is feasible and that $\widetilde{\Delta}^{\text{ci}}_1(\gamma) < \overline{\Delta}^p$. Then,

- if $\pi \in (\widetilde{\Delta}^{\text{ci}}_0(\gamma), \widetilde{\Delta}^{\text{ci}}_1(\gamma)]$ and $h(1) = 1$, I has an incentive to exit government after P invests; and
- if $\pi \in (\widetilde{\Delta}^{\text{ci}}_1(\gamma), \overline{\Delta}^p]$ and $h(1) = 1$, I has no incentive to exit government after P invests.

Proof. Suppose that competitive influence is feasible and that $\widetilde{\Delta}^{\text{ci}}_1(\gamma) < \overline{\Delta}^p$. The existence of a nonempty interval $(\widetilde{\Delta}^{\text{ci}}_0(\gamma), \widetilde{\Delta}^{\text{ci}}_1(\gamma)]$ follows directly from the fact that $\widetilde{\Delta}^{\text{ci}}_0(\gamma) < \widetilde{\Delta}^{\text{ci}}_1(\gamma)$ (see Lemma A13).

Next, we consider I's calculations if $\pi \in (\widetilde{\Delta}^{\text{ci}}_0(\gamma), \widetilde{\Delta}^{\text{ci}}_1(\gamma)]$ and $h(1) = 1$. By the definition of $\widetilde{\Delta}^{\text{ci}}_{h(0)}(\gamma)$, I gets more utility from being in government than not in this interval if $h(0) = 0$, but less utility from being in government than not if $h(0) = 1$. However, after P invests, then if $h(0) = 0$, I would benefit from exiting government. This follows from the fact that $\pi \leq \widetilde{\Delta}^{\text{ci}}_1(\gamma)$, and $\widetilde{\Delta}^{\text{ci}}_1(\gamma)$ characterizes the infimum of the set of πs required for I to find it optimal to enter government after P has invested.

Finally, we consider I's calculations if $\pi \in (\widetilde{\Delta}^{\text{ci}}_1(\gamma), \overline{\Delta}^p]$ and $h(1) = 1$. By the definition of $\widetilde{\Delta}^{\text{ci}}_{h(0)}(\gamma)$, I gets more utility from being in government than not in this interval for all $h(0) \in \{0, 1\}$. So, after P invests, I does not benefit from exiting government, which follows from the fact that $\pi > \widetilde{\Delta}_1^{\text{ci}}(\gamma)$, and $\widetilde{\Delta}_1^{\text{ci}}(\gamma)$ characterizes the infimum of the set of πs required for I to find it optimal to be in government after P has invested. □

With competitive influence, if $\pi > \widetilde{\Delta}_1^{\text{ci}}$, then we say the "strong entry condition" is satisfied since I has no incentive to exit government.

C.3 Retention Offer

We modify the game sequence slightly to allow for the possibility that G can offer a retention offer rq_G to I in exchange for her remaining employed with the group. I's utility is now defined over policy outcomes and the retention offer:

$$u_I^R = bq - \alpha_I(\hat{x}_G - x)^2 + pa(1-e)rq_G$$
$$u_G^R = bq - pa((1-e) + e\gamma)(c_G + (1-e)r)q_G - \alpha_G(\hat{x}_G - x)^2$$

Since we only modify the game to add one step at the beginning, the analysis of all subsequent subgames is as above. The issue requiring formal analysis is whether a retention offer is possible in equilibrium.

Label G's change in utility induced by I's entry in the main model by υ_G. Note that by Lemma A9, $\upsilon_G < 0$. Moreover, label the total retention pay $R = rq_G$

Proposition A3. If $|\upsilon_G^*| > \widetilde{R} > 0$ (where $\widetilde{R}$ is defined in the proof), then there exists a retention offer such that in an equilibrium, G successfully retains I with retention pay $R = \widetilde{R}$ and influences policy by proposing x_G^R and q_G^R (defined in proof), where $\hat{x}_P < x_G^R < x_G(0) < \hat{x}_G$. Otherwise, there is no equilibrium with successful retention, and the analysis is exactly as above.

Proof. With a successful retention offer (and given Assumption A2), G's optimal proposal is x_G^I and $\tilde{q}_G(x_G^I; h)$, where the analysis follows exactly as above, substituting c_G for $c_G + r$. Denote these policies as x_G^R and q_G^R, respectively. Assume G finds it optimal to offer a retention offer (implying it still influences policy after retaining I). Then I accepts any retention offer as long as:

$$(b+r)q_G^R - \alpha_I(\hat{x}_I - x_G^R)^2 \geq bq_G(1) - \alpha_I(\hat{x}_I - x(1))^2$$
$$\iff rq_G^R \geq \left[b(q(1) - q_G^R) - \alpha_I\left((\hat{x}_I - x(1))^2 - (\hat{x}_I - x_G^R)^2\right)\right] \equiv \widetilde{R}$$

If G seeks to retain I, then it offers just enough to retain I since its utility declines in r. Denote this offer as $\widetilde{R}$. We now analyze whether G finds it optimal to make an offer. As we showed above, G is worse off (in policy terms) when I enters. Denote this utility loss by $v_G < 0$. Moreover, it is only optimal for I to enter if she is better off doing so. So, if $|v_G| \geq \widetilde{R} > 0$, then G finds it optimal to make a retention offer $\widetilde{R}$ that will successfully retain I.

Finally, note that such retention offer raises G's marginal cost to produce quality, q_G. Examining the condition (1) from the proof of Lemma A2, it is obvious that x_G^I is lower with the retention offer than without it. □